TEMPERATURE DISTRIBUTION IN A LAMINAR STREAM OF LIQUID GIVING UP HEAT WHILE FLOWING IN A RECTANGULAR CHANNEL

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Temperature distributions, heat fluxes, and Nusselt numbers at the walls have been obtained for the case of laminar flow of a liquid in a rectangular channel for various laws of internal heat release.

In reference [2], for the case of laminar flow of an incompressible liquid in an infinite rectangular channel, the temperature distribution in the liquid and the heat flux through the wall were obtained with energy dissipation under constant channel wall temperature.

The present paper examines laminar flow of an imcompressible liquid in an infinite channel of rectangular cross section with arbitrary ratio of sides, with internal heat release in the liquid, described by relations of a different type. The basic assumptions are the same as in reference [2]: steady flow of the liquid is considered with constant physical properties, and the effect of gravity is not taken into account. The wall temperature is assumed constant and the same for all the channel walls.

Under the above assumptions, the differential equation of energy may be written in the following form [1]:

$$\frac{\partial^2 t}{\partial x^2}+\frac{\partial^2 t}{\partial y^2}=-\frac{Q(x, y)}{\lambda}. \tag{1}$$

The boundary conditions are

$$\begin{gathered} x=0;\quad x=a;\quad t=t_{\mathrm{w}};\\ y=0;\quad y=b;\quad t=t_{\mathrm{w}}. \end{gathered} \tag{2}$$

The heat release $Q(x, y)$ in the liquid is assumed to be representable in the form of a product of functions of the coordinates

$$Q(x, y)=\varphi(x)\cdot\psi(y).$$

A solution was obtained, following transition in (1) and (2) to the new variables

$$X=\frac{x}{a}\pi,\quad Y=\frac{y}{b}\pi,\quad T=t-t_{\mathrm{w}} \tag{3}$$

by use of a finite integral Fourier sine-transformation with respect to the variable X [4]. Quite a detailed similar solution has been examined in reference [2], and therefore we give here only the final results for the temperature fields over the channel section and for the heat fluxes at the walls.

The following cases of internal heat release in the liquid are examined:

$$Q(x, y)=Q_1=\mathrm{const}; \tag{4}$$

$$Q(x, y)=Q_2\sin\frac{x\pi}{a}; \tag{5}$$

$$Q(x, y)=Q_3\sin\frac{x\pi}{a}\sin\frac{y\pi}{b}; \tag{6}$$

$$Q(x, y)=Q_4\frac{x\pi}{a}. \tag{7}$$

The final expressions for the temperature field over the channel cross section and for the heat fluxes at the walls have the following form.

For the case $Q(x, y) = Q_1 = \mathrm{const}$

$$T=\frac{2}{\pi^3}\frac{a^2Q_1}{\lambda}\sum_{k=1}^{\infty}\frac{[1-(-1)^k]}{k^3}\times\left[1-\mathrm{ch}\left(kY\frac{b}{a}-k\frac{\pi b}{2a}\right]\Big/\mathrm{ch}\frac{k\pi b}{2a}\right]\sin kX; \tag{8}$$

$$q_{(x=0)}=-\lambda\left(\frac{\partial T}{\partial x}\right)_{x=0}=-\frac{2}{\pi^2}aQ_1\sum_{k=1}^{\infty}\frac{[1-(-1)^k]}{k^2}\times\left[1-\mathrm{ch}\left(kY\frac{b}{a}-k\pi\frac{b}{2a}\right)\Big/\mathrm{ch}\,k\pi\frac{b}{2a}\right]. \tag{9}$$

For the case $Q(x, y) = Q_2\sin(x\pi/a) = Q_2\sin X$

$$T=\frac{a^2}{\pi^2}\frac{Q_2}{\lambda}\left[1-\mathrm{ch}\left(Y\frac{b}{a}-\frac{\pi b}{2a}\right)\Big/\mathrm{ch}\frac{\pi b}{2a}\right]\sin X; \tag{10}$$

$$q_{(x=0)}=-\frac{aQ_2}{\pi}\left[1-\mathrm{ch}\left(Y\frac{b}{a}-\frac{\pi b}{2a}\right)\Big/\mathrm{ch}\frac{\pi b}{2a}\right]. \tag{11}$$

For the case $Q(x, y) = Q_3\sin(x\pi/a)\sin(y\pi/b) = Q_3\sin X\sin Y$

$$T=\frac{a^2}{\pi^2}\frac{Q_3}{\lambda}\sin Y\frac{\sin X}{1+a^2/b^2}; \tag{12}$$

$$q_{(x=0)}=-\frac{aQ_3}{\pi(1+a^2/b^2)}\sin Y. \tag{13}$$

For the case $Q(x, y) = Q_4x\pi/a = Q_4X$

$$T=-\frac{2}{\pi^2}\frac{Q_4a^2}{\lambda}\sum_{k=1}^{\infty}\frac{(-1)^k}{k^3}\left[1-\mathrm{ch}\left(kY\frac{b}{a}-k\frac{\pi b}{2a}\right)\Big/\mathrm{ch}\frac{k\pi b}{2a}\right]\sin kX; \tag{14}$$

$$q_{(x=0)}=\frac{2}{\pi}aQ_4\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2}\left[1-\mathrm{ch}\left(kY\frac{b}{a}-\frac{k\pi b}{2a}\right)\Big/\mathrm{ch}\frac{k\pi b}{2a}\right]. \tag{15}$$

Determination of Nusselt number at the walls was carried out as follows [3]:

$$\mathrm{Nu}=\frac{qd_{\mathrm{eq}}}{\lambda(t_{\mathrm{w}}-t_1)}, \tag{16}$$

where

$$d_{eq} = \frac{4F}{U}, \quad \bar{t}_1 = \int_0^\pi \int_0^\pi W_z t dX\, dY \Big/ \int_0^\pi \int_0^\pi W_z\, dX\, dY,$$

and the expression for W_z, obtained in reference [2], has the form

$$W_z = \frac{4A}{\pi} \sum_{p=1,3,5\ldots}^{\infty} \left[-1 + \mathrm{ch}\left(pY \frac{b}{a} - \frac{p\pi b}{2a} \right) \Big/ \mathrm{ch}\, \frac{p\pi b}{2a} \right] \frac{\sin pX}{p^3}. \quad (17)$$

Since the expressions for W_z and for T are in general infinite series, several terms were used for the calculations, depending on the rate of convergence of the corresponding series. The terms of the series decrease quite rapidly. For example, for each of the series described by (8) and (14) for $X = \pi/2$, the values of the third terms are less than 0.01 of the values of the first terms. In computing $\int_0^\pi \int_0^\pi t W_z dX dY$, where expressions (8) and (17) were used for T and W_z, it was sufficient to take a single term of each of the series (with $k = p = 1$), since after integration the value of the second term of the final expression was less than 0.002 of the value of the first term.

The results of the computations are shown in Figs. 1 and 2. All the computations were performed for the same values of internal heat release per unit channel length, i. e., with

$$Q_2 = \frac{\pi}{2} Q_1, \quad Q_3 = \frac{\pi^2}{4} Q_1, \quad Q_4 = \frac{2}{\pi} Q_1.$$

Figure 1a shows curves of temperature distribution in dimensionless form $T/(a^2 Q_1/\lambda)$, obtained according to the formulas given above. Curve 5 was constructed for the case of heat release in the liquid with the law $Q(x, y) = Q_5 (1 - \sin(x\pi/a)) = Q_5 (1 - \sin X)$, and was obtained by combining solutions of Eq. (1) with $Q(x, y) = Q_1$ and $Q(x, y) = Q_2 \sin X$. Here,

$$Q_5 = \frac{\pi}{\pi - 2} Q_1.$$

It may be seen from Fig. 1a that the most uniform temperature field across the section with a symmetrical distribution of heat sources relative to the mean plane of the channel occurs when the maxima of heat release are displaced as close as possible to the walls, and, conversely, the most nonuniform temperature field with the greatest heating of the liquid relative to the wall is obtained for heat release maxima close to the channel axis. The heating of the liquid at the center of the channel, with a heat release distribution according to the law $Q = Q_5 (1 - \sin X)$, is approximately four times less than with a heat release law $Q = Q_3 \sin X \sin Y$, and less by a factor of approximately three than for a heat release law $Q = Q_2 \sin X$ under the same heat release per unit channel length in all cases.

With an asymmetrical heat release, the temperature field also remains asymmetrical (curve 4), and a corresponding redistribution of heat fluxes at the walls takes place.

Figure 1b shows curves of temperature distribution in the mean plane $X = \pi/2$ of the channel with ratio of sides $b/a = 2$. The general picture shows almost no change from that of Fig. 1. The minimum temperature of the channel axis—for heat release law $Q = Q_5(1 - \sin X)$—is also less by a factor of approximately 4 than for a heat release law $Q = Q_3 \sin X \sin Y$. It is interesting that the temperature distribution for a heat release law $Q = Q_1$ coincides with the temperature distribution for a heat release law $Q = Q_4$ (curves 1 and 4). The heat fluxes at the walls for $X = \pi/2$ are quite considerably different from those with $Y = \pi/2$.

The distribution of Nusselt number at the walls for a channel of square section, as obtained by formula (16), is shown in Fig. 2. Since only q is a function of X and Y in formula (16), the curves in Fig. 2 describe the distribution of heat flux at the walls in the appropriate scale.

NOTATION

V_z is the stream velocity; x, y are the coordinates perpendicular to the stream direction; Q(x, y) is the internal heat release; λ is the thermal conductivity of the liquid; t is the temperature; a, b are width and height of the channel; q is the specific heat flux at the wall; d_{eq} is the equivalent diameter; a is the a constant; F, U are the area and perimeter of the channel; t_W is the wall temperature; $\bar{t}_l$ is the mean enthalpy temperature of the liquid.

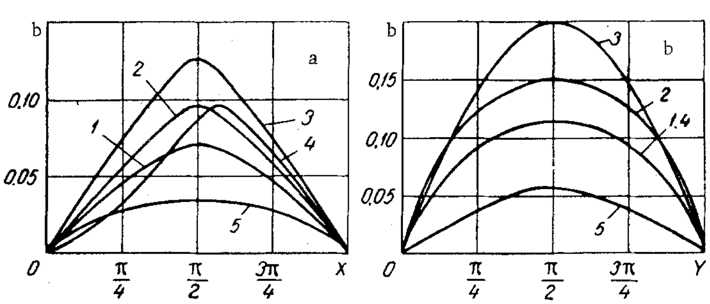

Fig. 1. Temperature distribution in the mean plane $Y = \pi/2$ of a channel of square section (a), and in the mean plane $(X = \pi/2)$ of a channel with ratio of sides $b/a = 2$ (b): 1—for $Q = Q_1$; 2—$Q = Q_1(\pi/2) \sin X$; 3—$Q = Q_1(\pi^2/4) \sin X \cdot \sin Y$; 4—$Q = Q_1(2/\pi)X$; 5—$Q = Q_1[\pi/(\pi - 2)](1 - \sin X)$; $B = T/(a^2 Q_1/\lambda)$.

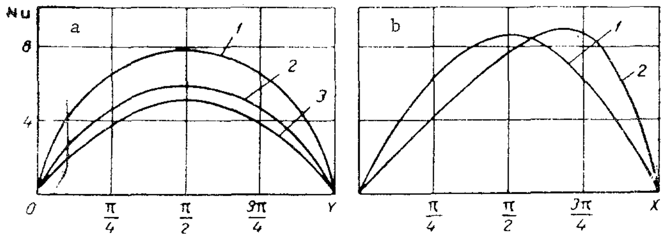

Fig. 2. Nusselt number distribution for a square section channel: a – at the wall (X = 0) with $Q = Q_1$ (1), $Q = Q_1 (\pi/2) \sin X$ (2) and $Q = Q_1 (\pi^2/2) \sin X \sin Y$ (3); b – at the wall (Y = 0) with $Q = Q_1 (\pi/2) \sin X$ (1) and $Q = Q_1 (2/\pi) X$ (2).

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